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A New Class of Ternary Zero Correlation Zone Sequence Sets Based on Mutually Orthogonal Complementary Sets

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Abstract: *This paper proposes a new class of ternary zero correlation zone (ZCZ) sequence sets based on mutually orthogonal complementary sets (MOCS), in which the periodic cross-correlation function and the out-of-phase of the periodic auto-correlation function of the proposed sequence set are zero in a specified zone of phase shift. It is shown that the proposed zero correlation zone sequence set can achieve the upper bound on the ternary ZCZ codes.*

Keywords - *Sequence design, mutually orthogonal complementary sets, theoretical upper bound, zero correlation zone (ZCZ) sequences*.

I. Introduction

Different types of sequence set used in communications systems have been studied in order to reduce the Multiple Access Interference (MAI) [1-6].

Zero correlation zone (ZCZ) sequences, which have vigorously been studied now, are defined to be a sequence set with ZCZ which means the duration with zero auto-correlation function and zero cross-correlation function at out-of-phase state [1-12]. When ZCZ sequences are used as spreading sequences in DS-CDMA, it can effectively eliminate MAI if all multiple access delays are inside ZCZ [1-6]. There are various intensive studies on the constructing of ZCZ sequences including binary, ternary and polyphase sequences [1-12].

Generally, sets of ZCZ sequences are characterized by the period of sequences N, the family size, namely the number of sequences M, and the length of the zero-correlation zone Z_{CZ} [7]. A ZCZ (N, M, Z_{CZ}) sequence set that satisfies the theoretical bound defined by the ratio $M(Z_{cz}+1)/N = 1$ is called an optimal zero-correlation zone sequence set [7, 10].

In this paper, we propose a new construction method to obtain ternary ZCZ sequence sets based on binary mutually orthogonal complementary set (MOCS) and from padding zero between sequences of MOCS. Our proposed ternary zero-correlation zone sequence set is almost optimal ZCZ sequence set.

The paper is organized as follows. Section 2 introduces the notations required for the subsequent sections, the proposed scheme for sequence construction is explained in section 3. Examples of new ZCZ sequence sets are presented in Section 4. The properties of the proposed sequence sets are shown in Section 5. Finally, we draw the concluding remarks.

II. Notations

2.1 Definition 1: Suppose X_j=$(x_{j,0}, x_{j,1}, \dots\dots . x_{j,N-1})$ and, X_v=$(x_{v,0}, x_{v,1}, \dots\dots . x_{v,N-1})$ are two sequences of period N. Sequence pair (X_j, X_v) is called a binary sequence pair if
$x_{j,i}, x_{v,i} \in \{-1,+1\}$, $i = 0,1,2, \dots\dots\dots N-1$ [2].

The Periodic Correlation Function (PCF) between X_j and X_v at a shift τ is defined by [8]:

$$\forall\tau \geq 0, \theta_{(X_j, X_v)}(\tau) = \sum_{i=0}^{N-1} x_{j,i} x_{v,(i+\tau)\mathrm{mod}\,(N)} \quad \text{and} \quad \theta_{(X_v, X_j)}(-\tau) = \theta_{(X_j, X_v)}(\tau) \qquad (1)$$

2.2 Definition 2: A set of M sequences $\{X_0, X_1, X_2, \dots\dots, X_{M-1}\}$ is denoted by $\{X_j\}_{j=0}^{M-1}$.
A set of sequences $\{X_j\}_{j=0}^{M-1}$ is called zero correlation zone sequence set, denoted by Z(N,M,Z_{CZ}) if the periodic correlation functions satisfy [8] :

$$\forall j, 0 < |\tau| \leq Z_{cz}, \theta_{(X_j, X_j)}(\tau)=0 \qquad (2)$$
$$\forall j, (j \neq v), |\tau| \leq Z_{cz}, \theta_{(X_j, X_v)}(\tau)=0 \qquad (3)$$

2.3 Definition 3: Let each element of an $H \times H$ matrix $F^{(n)}$ ($n \geq 0$) be a sequence of length $l = 2^n \times l_m =$ 2m+n, where lm=2m and m≥0 [5].

$$F^{(n)} = \begin{bmatrix} F_{11}^{(n)} & F_{12}^{(n)} & \cdots & F_{1k}^{(n)} & F_{1H}^{(n)} \\ F_{21}^{(n)} & F_{22}^{(n)} & & F_{2k}^{(n)} & F_{2H}^{(n)} \\ \vdots & & \ddots & & \vdots \\ F_{H1}^{(n)} & F_{H2}^{(n)} & \cdots & F_{Hk}^{(n)} & F_{HH}^{(n)} \end{bmatrix}_{(H\times H)} \quad (4)$$

If F satisfies the following formulas, then F is a MOCS [13]:

$$\sum_{k=1}^{H} \varphi_{F_{j,k}\,F_{j,k}}(\tau) = 0, \qquad \text{for } \forall j, \forall\tau \neq 0 \quad (5)$$

$$\sum_{k=1}^{H} \varphi_{F_{j,k}F_{s,k}}(\tau) = 0, \ \text{for } \forall j \neq s\,, \forall\tau \quad (6)$$

Where $\varphi_{F_{j,k}\,F_{j,k}}(\tau)$ and $\varphi_{F_{j,k}F_{s,k}}(\tau)$ are the aperiodic autocorrelation and cross-correlation functions, respectively [10].

III. Proposed Sequence Construction

In this section, a new method for constructing sets of ternary ZCZ sequences is proposed. The construction is accomplished through three steps.

3.1 Step 1: The matrix MOCS of dimension $(H \times H) = (2^{n+1}, 2^{n+1})$. Since each element in $F^{(n)}$ is a sequence of length $l = 2^n \times l_m = 2^{m+n}$, each raw has a length of $L = 2^{2n+m+1}$. The matrix MOCS is $(2^{n+1} \times 2^{2n+m+1})$ with $n \geq 0$ and $m \geq 0$ [5].

$$F^{(n)} = \begin{bmatrix} F_{11}^{(n)} & F_{12}^{(n)} & \cdots & F_{1k}^{(n)} & F_{1H}^{(n)} \\ F_{21}^{(n)} & F_{22}^{(n)} & & F_{2k}^{(n)} & F_{2H}^{(n)} \\ \vdots & & \ddots & & \vdots \\ F_{H1}^{(n)} & F_{H2}^{(n)} & \cdots & F_{Hk}^{(n)} & F_{HH}^{(n)} \end{bmatrix} = \begin{bmatrix} f_1 \\ \vdots \\ f_j \\ \vdots \\ f_H \end{bmatrix} \quad (7)$$

A set of $M = 2H = 2^{n+2}$ sequences d_j, each of length $S = (2L+2) = 2^{2n+m+2} + 2$, is constructed as follows:

For $1 \leq j \leq H$

$$d_{j+0} = [-f_j\ 0\ f_j\ 0] = [-F_{j1}^{(n)} \ldots -F_{jk}^{(n)} \ldots . -F_{jH}^{(n)} 0\ F_{j1}^{(n)} \ldots F_{jk}^{(n)} \ldots .\ F_{jH}^{(n)} 0] \quad (8)$$

$$d_{j+1} = [f_j\ 0\ f_j\ 0] = [F_{j1}^{(n)} \ldots F_{jk}^{(n)} \ldots .\ F_{jH}^{(n)} 0\ F_{j1}^{(n)} \ldots F_{jk}^{(n)} \ldots .\ F_{jH}^{(n)} 0] \quad (9)$$

Where $[f_j 0\ f_j\ 0]$ is called the concatenation operation between two rows f_j and f_j of the matrix $F^{(n)}$ and two zeros.

3.2 Step 2: For the first iteration, $p = 0$, we can generate , based on interleaving technique in [10], a series of sets $\{T_j\}$ of M sequences.

A pair of sequences T_{j+0} and T_{j+1} of length $N = (2^{p+1} \times S) = (2^{p+1} \times (2^{2n+m+2} + 2)) = (2^{p+1} \times 2\times L+2$ is constructed by interleaving elements ($\pm Fjkn\ et\ 0$) of a sequence pair $dj+0$ and $dj+1$ as follows (by using equations (8) and (9)):

For $1 \leq j \leq H$,

$$T_{j+0} = [d_{j+0,1}, d_{j+1,1}, d_{j+0,2}, d_{j+1,2}, \ldots \ldots, d_{j+0,S}, d_{j+1,S}] \quad (10)$$

$$T_{j+0} = [-F_{j1}^{(n)}\ F_{j1}^{(n)} \ldots - F_{jk}^{(n)}\ F_{jk}^{(n)} \ \ldots\ -F_{jH}^{(n)}\ F_{jH}^{(n)}\ 0\ 0\ F_{j1}^{(n)}\ F_{j1}^{(n)}\ \ldots F_{jk}^{(n)} F_{jk}^{(n)} \ldots\ F_{jH}^{(n)}\ F_{jH}^{(n)}\ 0\ 0]$$

and,

$$T_{j+1} = [d_{j+0,1}, -d_{j+1,1}, d_{j+0,2}, -d_{j+1,2}, \ldots \ldots, d_{j+0,S}, -d_{j+1,S}] \quad (11)$$

$$T_{j+1} = [-F_{j1}^{(n)} - F_{j1}^{(n)} \ldots - F_{jk}^{(n)} - F_{jk}^{(n)} \ldots - F_{jH}^{(n)} - F_{jH}^{(n)}\ 0\ 0\ F_{j1}^{(n)} - F_{j1}^{(n)} \ldots\ F_{jk}^{(n)} - F_{jk}^{(n)} \ldots\ F_{jH}^{(n)} - F_{jH}^{(n)} 0\ 0\ ]$$

The member size of the sequence set $\{T_j\}$ is $M = 2H = 2^{n+2}$.

3.3 Step 3: For $p > 0$, we can recursively construct a new series of set, $\{T_j\}$, by interleaving of actual $\{T_j\}$ (in equations (10) and (11)). Both sequences T_{j+0} and T_{j+1} are of length $N = (2^{p+1} \times S)$.

The $\{T_j\}$ is generated as follows:
For $1 \leq j \leq H$,

$$T_{j+0} = [T_{j+0,1}, T_{j+1,1}, T_{j+0,2}, T_{j+1,2}, \dots \dots, T_{j+0,2S}, T_{j+1,2S}] \tag{12}$$
$$T_{j+0} = \left[-F_{j1}^{(n)} - F_{j1}^{(n)} F_{j1}^{(n)} - F_{j1}^{(n)} \dots - F_{jk}^{(n)} - F_{jk}^{(n)} F_{jk}^{(n)} - F_{jk}^{(n)} \dots - F_{jH}^{(n)} - F_{jH}^{(n)} F_{jH}^{(n)} - F_{jH}^{(n)} 0000 F_{j1}^{(n)} F_{j1}^{(n)} F_{j1}^{(n)} - F_{j1}^{(n)} \dots F_{jk}^{(n)} F_{jk}^{(n)} F_{jk}^{(n)} - F_{jk}^{(n)} \dots F_{jH}^{(n)} F_{jH}^{(n)} F_{jH}^{(n)} - F_{jH}^{(n)} 0000\right]$$

and

$$T_{j+1} = [T_{j+0,1}, -T_{j+1,1}, T_{j+0,2}, -T_{j+1,2}, \dots \dots, T_{j+0,2S}, -T_{j+1,2S}] \tag{13}$$
$$T_{j+1} = [-F_{j1}^{(n)} F_{j1}^{(n)} F_{j1}^{(n)} F_{j1}^{(n)} \dots - F_{jk}^{(n)} F_{jk}^{(n)} F_{jk}^{(n)} F_{jk}^{(n)} \dots - F_{jH}^{(n)} F_{jH}^{(n)} F_{jH}^{(n)} F_{jH}^{(n)} 0000 F_{j1}^{(n)} - F_{j1}^{(n)} F_{j1}^{(n)} F_{j1}^{(n)} \dots F_{jk}^{(n)} - F_{jk}^{(n)} F_{jk}^{(n)} F_{jk}^{(n)} \dots F_{jH}^{(n)} - F_{jH}^{(n)} F_{jH}^{(n)} F_{jH}^{(n)} 0000]$$

IV. Example Of Construction

4.1 Step 1: Let $F^{(1)}$ a matrix MOCS $(H \times L) = (4 \times 8), n = 1, m = 0, l = 2^n \times l_m = 2^{m+n} = 2$.

$$F^{(1)} = \begin{bmatrix} F_{11}^{(1)} & F_{12}^{(1)} & F_{13}^{(1)} & F_{14}^{(1)} \\ F_{21}^{(1)} & F_{22}^{(1)} & F_{23}^{(1)} & F_{24}^{(1)} \\ F_{31}^{(1)} & F_{32}^{(1)} & F_{33}^{(1)} & F_{34}^{(1)} \\ F_{41}^{(1)} & F_{42}^{(1)} & F_{43}^{(1)} & F_{44}^{(1)} \end{bmatrix} = \begin{bmatrix} f_1 \\ \vdots \\ f_j \\ \vdots \\ f_4 \end{bmatrix}$$

$$F^{(1)} = \begin{bmatrix} -- & ++ & +- & -+ \\ -- & -- & +- & +- \\ +- & -+ & -- & ++ \\ +- & +- & -- & -- \end{bmatrix}$$

Where (-) and (+) are (-1) and (+1), respectively.
A set of $M = 8$ sequences d_j , each of length $S = 18$, is constructed as follows (using equations (8) and (9)):
For $1 \leq j \leq 4$,
$d_{1+0} = [-f_1\, 0, f_1 0]$
$= \left[-F_{11}^{(1)} - F_{12}^{(1)} - F_{13}^{(1)} - F_{14}^{(1)} 0 F_{11}^{(1)} F_{12}^{(1)} F_{13}^{(1)} F_{14}^{(1)} 0\right] = [+ + \; - - \; - + \; + - 0 - - \; + + \; + - \; - + 0]$
It follows for the other sequences:
$d_{2+0} = [+ + \; + + \; - + \; - + 0 \; - - \; - - \; + - \; + - 0]$
$d_{3+0} = [- + \; + - \; + + \; - - 0 \; + - \; - + \; - - \; + + 0]$
$d_{4+0} = [+ - \; - + \; + + \; + + 0 \; + - \; + - \; - - \; - - 0]$
and,
$d_{1+1} = [f_1\, 0, f_1 0]$
$= \left[F_{11}^{(1)} F_{12}^{(1)} F_{13}^{(1)} F_{14}^{(1)} 0 F_{11}^{(1)} F_{12}^{(1)} F_{13}^{(1)} F_{14}^{(1)} 0\right] = [- - \; + + \; + - \; - + 0 \; - - \; + + \; + - \; - + 0]$
$d_{2+1} = [- - \; - - \; + - \; + - 0 \; - - \; - - \; + - \; + - 0]$
$d_{3+1} = [+ - \; - + \; - - \; + + 0 \; + - \; - + \; - - \; + + 0]$
$d_{4+1} = [+ - \; + - \; - - \; - - \; 0 + - \; + - \; - - \; - - 0]$

4.2 Step 2: For the first iteration, $p = 0$.
A pair of sequences T_{j+0} and T_{j+1} of length $N = (2^{p+1}S) = 36$ is constructed as follows (using equations (10) and (11)):
For $1 \leq j \leq 4$,
$T_{1+0} = [+ + \; - - \; - - \; + + \; - + \; + - \; + - \; - + 00 \; - - \; - - \; + + \; + + \; + - \; + - \; - + \; - + 00]$
$T_{2+0} = [+ + \; - - \; + + \; - - \; - + \; + - - + \; + - \; 00 - - \; - - \; - - \; - - \; + - \; + - \; + - \; + - \; 00]$
$T_{3+0} = [- + \; + - \; + - \; - + \; + + - \; - \; - - \; + + 00 + - \; + - \; - + \; - + \; - - \; - - \; + + \; + + 00]$
$T_{4+0} = [- + \; + - \; - + \; + - \; + + \; - - \; + + \; - - 00 \; + - \; + - \; + - \; + - \; - - \; - - \; - \; - \; - - 00]$
$T_{1+1} = [+ + \; + + \; - - \; - - - + \; - + \; + - \; + - \; 00 - - \; + + \; + + \; - - \; + - \; - + \; - + \; + - 00]$
$T_{2+1} = [+ + \; + + \; + + \; + + \; - + \; - + \; - + \; - + 00 \; - - \; + + \; - - \; + + \; + - \; - + \; + - \; - + 00]$
$T_{3+1} = [- + \; - + \; + - \; + - \; + + \; + + \; - - \; - - 00 \; + - \; - + \; - + \; + - \; - - \; + + \; + + \; - - 00]$
$T_{4+1} = [- + \; - + \; - + \; - + \; + + \; + + \; + + \; + + 00 \; + - \; - + \; + - \; - + \; - - \; + + \; - \; - \; + + 00]$

4.3 Step 3: For the next iteration $p = 1$, the $\{T_j\}$ is generated as follows (using equations (12) and (13)):

T_{1+0} = [++ ++ −− ++ −− −− ++ −− −+ −+ +− −+ +− +− −+ +− 0000 − −− −
− ++ ++ ++ ++ −− +− +− +− −+ −+ −+ −+ +− 0000]

T_{2+0} = [++ ++ −− ++ ++ ++ −− ++ −+ −++− −+ −+ −+ +− −+ 0000 −− −−
−− ++ −− −− −− ++ +− +− +− −+ +− +− +− −+0000]

T_{3+0} = [−+ −+ +− −+ +− +− −+ +− ++ ++ −− ++ −− −− ++ −− 0000 +− +
− +− −   + − + +− −+ −+ −− +− −− −− ++ ++ ++ −−0000]

T_{4+0} = [−+ −+ +− −+ −+ −+ +− −+ ++ ++ −− ++ ++ ++ −− ++ 0000 +− +−
+− −+ +− +− +− −+ −− −− −− ++ −− −− −− ++0000]

T_{1+1} = [++ −− −− −− −− ++ ++ ++ −+ +− +− +− +− −+ −+ −+ 0000 −− ++
−− −− ++ −− ++ ++ +− −+ +− +− −+ +− −+ −+ 0000]

T_{2+1} = [++ −− −− −− ++ −− −− −− −+ +− +− +− −+ + −+ −+ − 0000 −− ++
−− −− −− ++ −− −− +− −+ +− +− +− −+ +− +−0000]

T_{3+1} = [−+ +− +− +− +− −+ −+ −+ + +− − −− −− −− ++ ++ ++ 0000 +− −+
+− +− −+ +− −+ −+ −− ++ −− −− ++ −− ++ ++0000]

T_{4+1} = [−+ +− +− +− −+ +− +− +− ++ −− −− −− ++ −− −− −− 0000 +− −+
+− +− +− −+ +− +− −− ++ −− −− −− ++ −− −−0000]

The length of both sequences T_{j+0} and T_{j+1} is equal to $N = (2^{p+1}S) = 72$.
The member size of the sequence set $\{T_j\}$ is $M = 8$.

Fig. 1 shows the auto-correlation function given in Equation (1) of T_{1+0}, and **Fig. 2** shows the cross-correlation function given in Equation (1) of T_{1+0} with T_{2+0}.

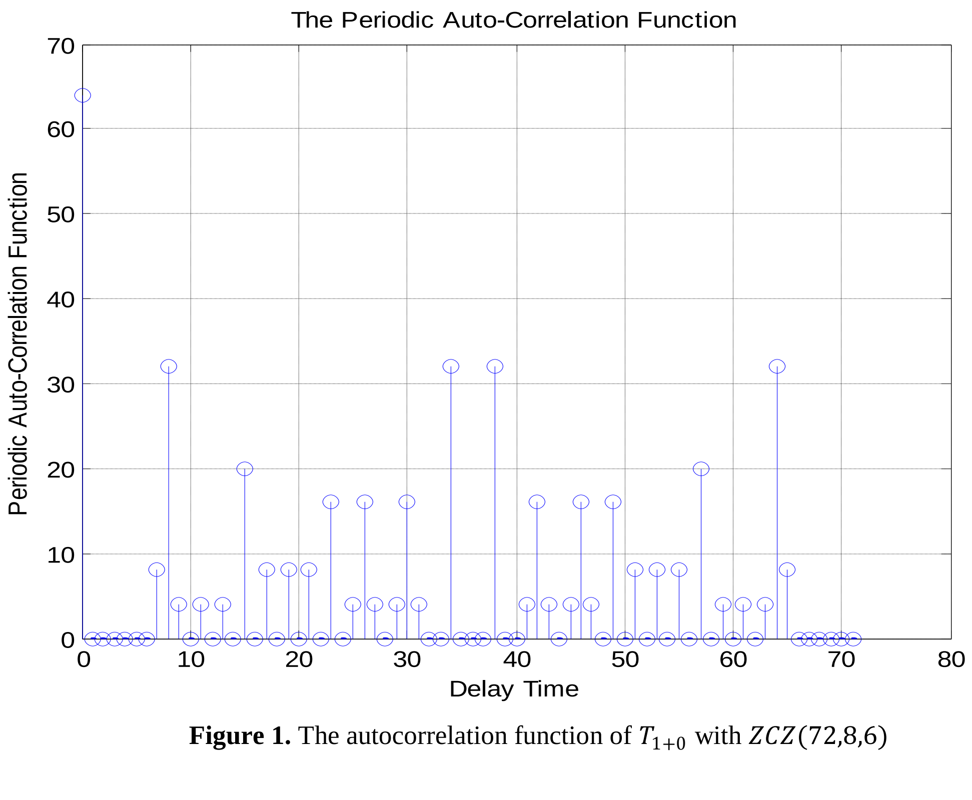

Figure 1. The autocorrelation function of T_{1+0} with $ZCZ(72,8,6)$

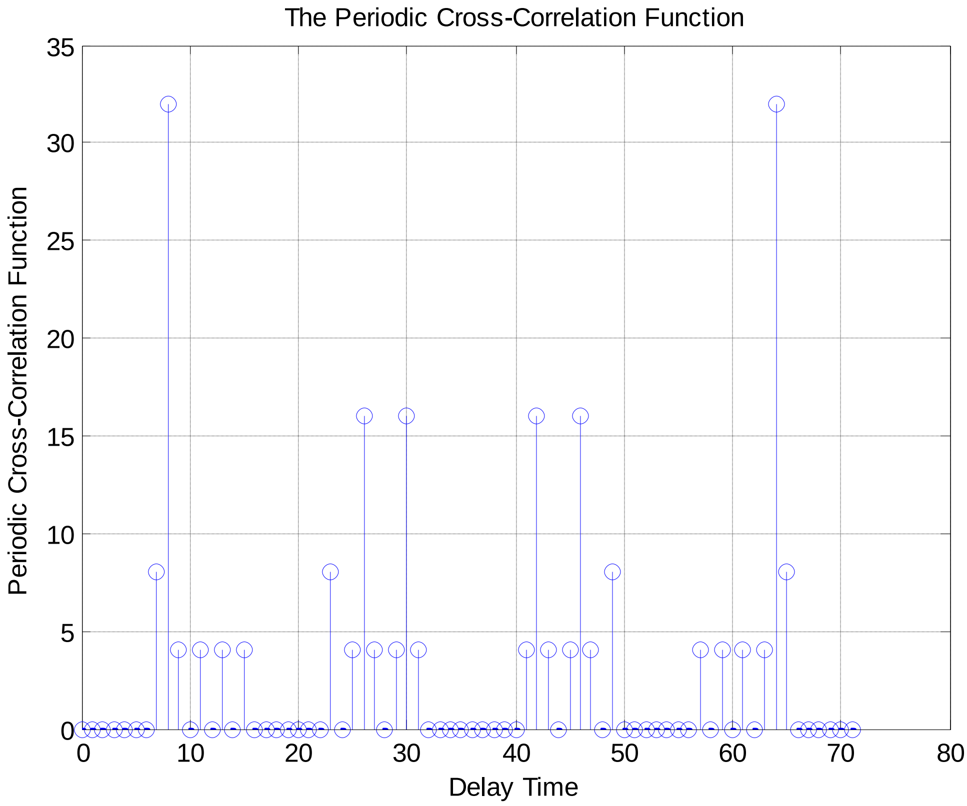

Figure 2. The cross-correlation function between T_{1+0} and T_{2+0} with $ZCZ(72,8,6)$

The periodic correlation function confirm that $\{T_j\}$ is a ZCZ (72,8,6) sequence set.

V. The Properties Of The Proposed Sequence

The proposed ternary ZCZ sequence set can satisfy the ideal autocorrelation and cross-correlation properties in the zero-correlation zone.

The generated sequence set satisfies the following properties:

$\forall j, \forall \tau \neq 0, |\tau| \leq Z_{CZ}$

$\theta_{(T_j, T_j)}(\tau)=0$ (14)

and,

$\forall j \neq v, \forall \tau, |\tau| \leq Z_{CZ}$

$\theta_{(T_j, T_v)}(\tau)=0.$ (15)

The $\{T_j\}$ is a ternary ZCZ sequence set having parameters:

1) For $n+m=0$, $ZCZ(N, M, Z_{CZ}) = ZCZ(2^{p+2}(1+2^{2n+m+1}), 2^{n+2}, 2^{n+m+p+1}-1) = ZCZ(2^{p+1} \times 6,4,2p+1-1$.

2) For $n+m=1$, $ZCZ(N, M, Z_{CZ}) = ZCZ(2^{p+2}(1+2^{2n+m+1}), 2^{n+2}, 2^{n+m+p+1}-2) = ZCZ(2^{p+2}(1+2n+2,2n+2,2p+2-2$.

3) For $n+m>1$, $ZCZ(N, M, Z_{CZ}) = ZCZ(2^{p+2}(1+2^{2n+m+1}), 2^{n+2}, 2^{n+m+p}+4 \times p)$.

VI. Conclusion

In this paper, we have proposed a new method for constructing sets of ternary ZCZ sequences based on binary mutually orthogonal complementary set (MOCS) and from padding zero between sequences of MOCS. The periodic auto-correlation function side lobes and the periodic cross-correlation function of the proposed sequence set is zero for the phase shifts within the zero correlation zone. The proposed ternary ZCZ sequence set is almost optimal ZCZ sequence set.

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